

## ON CHARACTERIZATION OF MINIMAL $k$ -BI-IDEALS IN $k$ -REGULAR AND COMPLETELY $k$ -REGULAR SEMIRINGS

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**Abstract.** In this paper, we study  $k$ -regular and completely  $k$ -regular semirings. We characterize the minimal  $k$ -bi-ideals in  $k$ -regular semirings via principal  $k$ -bi-ideals and also in completely  $k$ -regular semirings via  $k$ -bi-ideals generated by  $k$ -idempotent elements. Finally we characterize the completely  $k$ -regular semirings by  $k$ -bi-ideals generated via  $k$ -idempotents.

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### 1. Introduction

The notion of a semiring was introduced by Vandiver [15]. In 1951, Bourne defined a regular semiring as a semiring  $S$  in which for all  $a \in S$  there exist  $x, y \in S$  such that  $a + axa = aya$ . In [1], Adhikari, Sen and Weinert renamed it as a  $k$ -regular semiring. In [14], Sen and Bhuniya studied  $k$ -regular semirings with a semilattice additive reduct, and constructed  $k$ -regular semirings. If  $F$  is any semigroup, then the set  $P(F)$  of all subsets of  $F$  is a semiring in  $\mathcal{S}\mathcal{L}^+$ , where addition and multiplication are defined by the set union and the usual product of subsets of a semigroup, respectively. In [14], it is shown that  $P(F)$  is a  $k$ -regular semiring if and only if  $F$  is a regular semigroup [Theorem 3.1], and if  $(F, \cdot)$  is a regular semigroup, then the  $k$ -idempotents of  $P(F)$  commute if and only if  $P(F)$  is a commutative semiring [Theorem 3.4]. Sen and Bhuniya defined  $k$ -idempotents to characterize the  $k$ -regular semirings which are distributive lattices of  $k$ -semifields [13]. Bhuniya and Jana introduced the notion of  $k$ -bi-ideals in a semiring, characterized the  $k$ -regular semirings by  $k$ -bi-ideals, and gave the description of the principal  $k$ -bi-ideals in a semiring with semilattice additive reduct [2]. In [9], Jana studied quasi  $k$ -ideals in  $k$ -regular semirings and characterized the  $k$ -regular semirings via their quasi  $k$ -ideals. In [12], Sen and Bhuniya defined completely  $k$ -regular semirings and presented various interesting properties of classes of such semirings. They characterized

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